Towards a Linear State-Space Model of a Wind Power Plant

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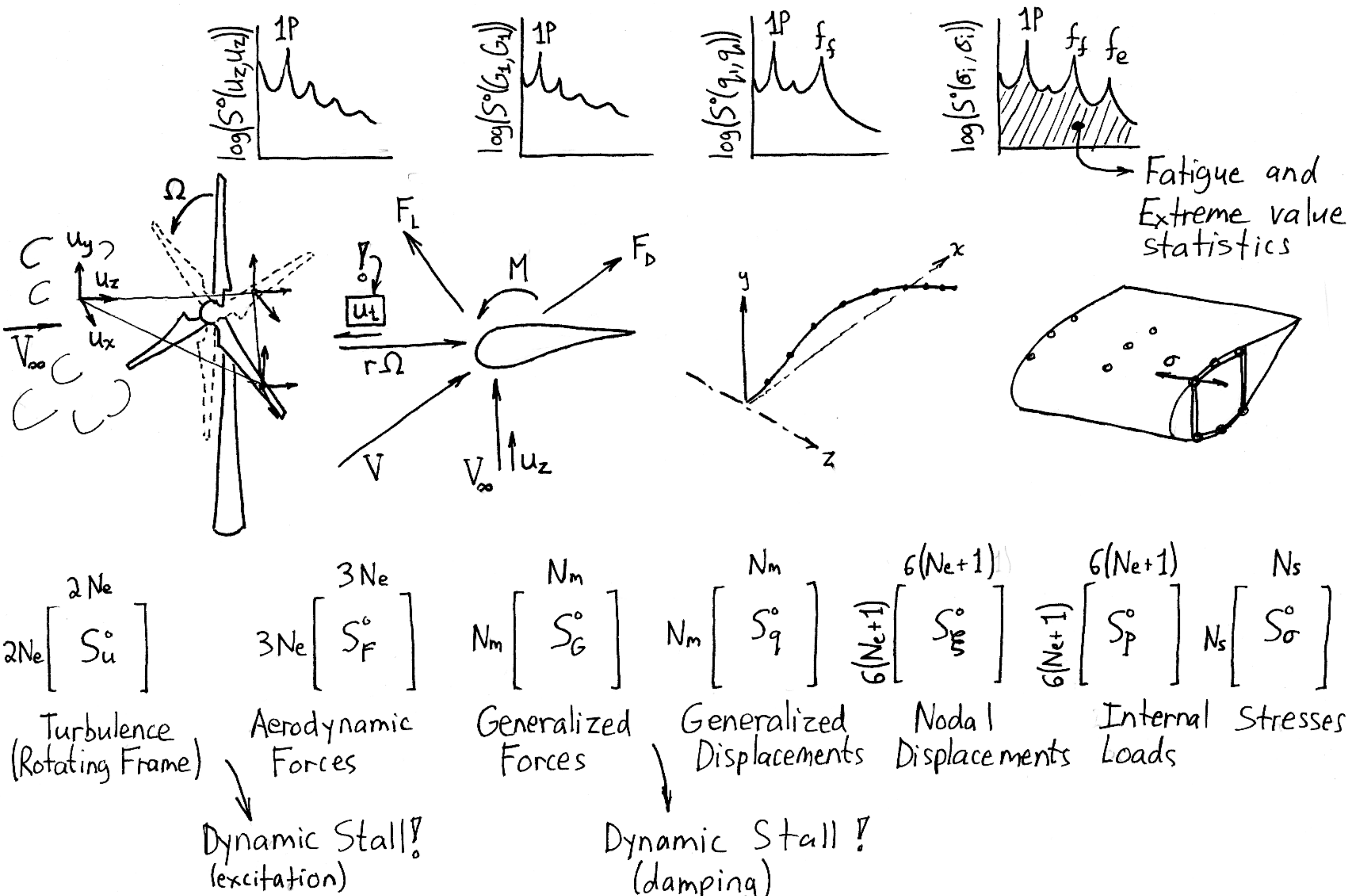
Frequency-domain analysis

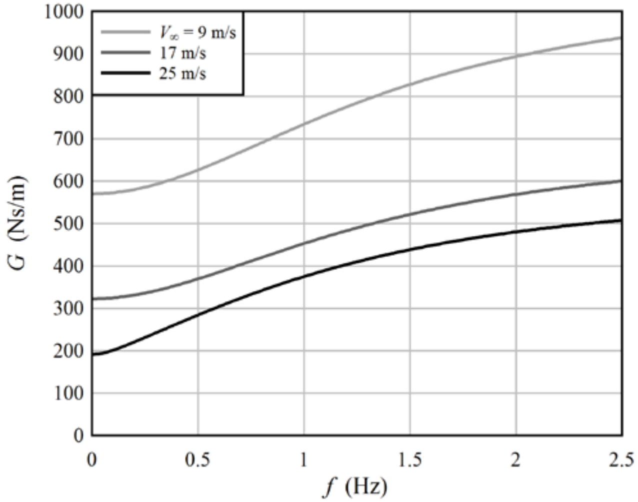


(TURBU, HAWCStab, STAS...)

Solving a linear time-lag equation (similar to Øye method):

$$\left(\frac{dC_L}{d\alpha}\right)_{\text{in}} = \left(\frac{dC_L}{d\alpha}\right)_{\text{max}} + \left[\left(\frac{dC_L}{d\alpha}\right)_{\text{q}} - \left(\frac{dC_L}{d\alpha}\right)_{\text{max}}\right]\left(\frac{1}{1+(\tau\omega)^2}\right)$$

$$\left(\frac{dC_L}{d\alpha}\right)_{\text{out}} = \left[\left(\frac{dC_L}{d\alpha}\right)_{\text{max}} - \left(\frac{dC_L}{d\alpha}\right)_{\text{q}}\right]\left(\frac{\tau\omega}{1+(\tau\omega)^2}\right)$$



Excitation by turbulence:

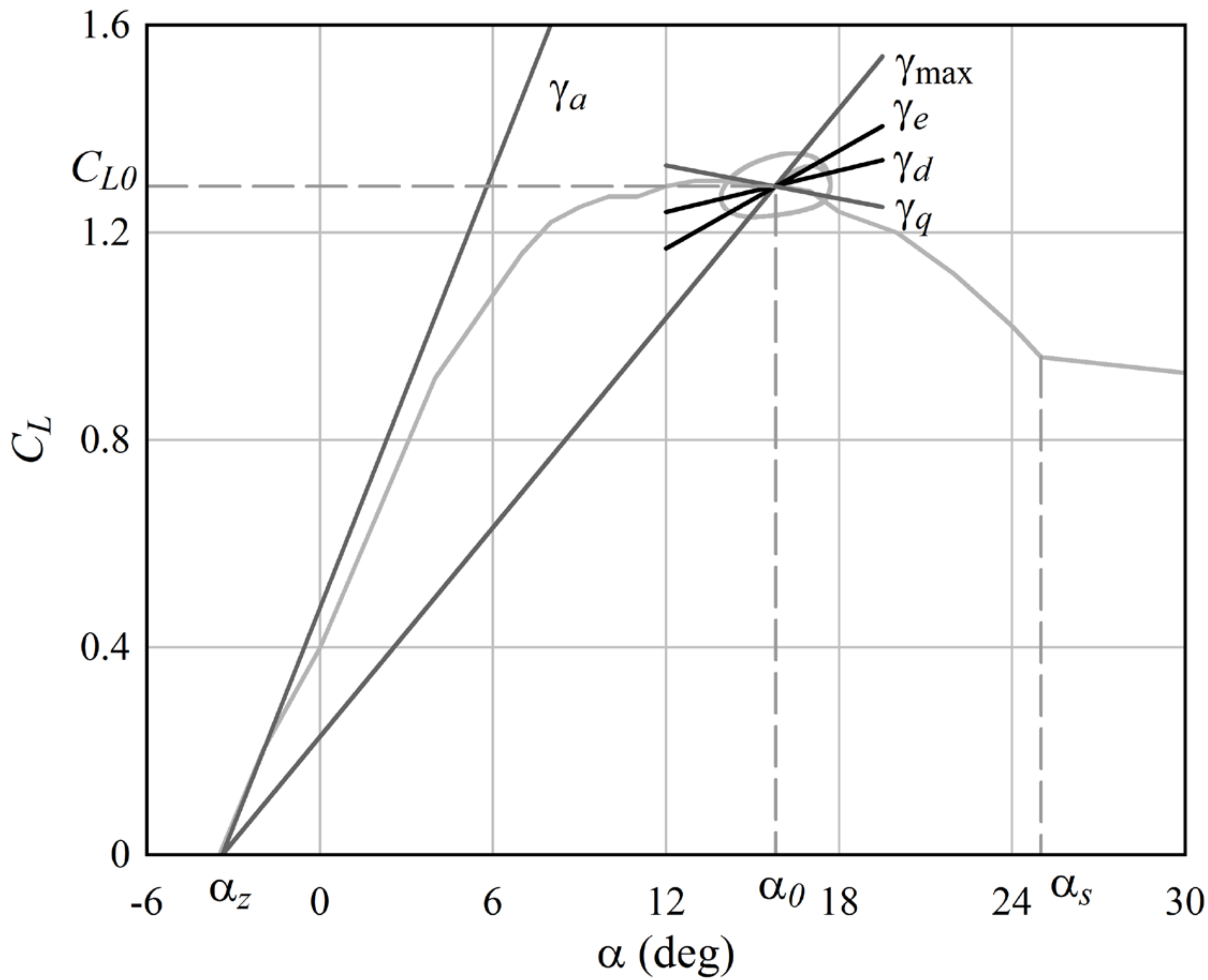
$$\left|\frac{dF_y}{du_z}\right|_e = \left|\frac{1}{2}\rho c L\left[\frac{2\left(V_\infty + V_{i,z}\right)^2 + (r\Omega)^2}{|V_0|}C_L + r\Omega\frac{V_\infty + V_{i,z}}{|V_0|}\left(\frac{dC_L}{d\alpha}\right)_{\text{in}} - r\Omega\frac{V_\infty + V_{i,z}}{|V_0|}C_D - \frac{(r\Omega)^2}{|V_0|}\frac{dC_D}{d\alpha}\right] + i\frac{1}{2}\rho c L\left[r\Omega\frac{V_\infty + V_{i,z}}{|V_0|}\left(\frac{dC_L}{d\alpha}\right)_{\text{out}}\right]\right|$$

Aerodynamic damping:

$$\frac{dF_y}{du_z} = \frac{1}{2}\rho c L\left[\frac{2\left(V_\infty + V_{i,z}\right)^2 + (r\Omega)^2}{|V_0|}C_L + r\Omega\frac{V_\infty + V_{i,z}}{|V_0|}\left(\frac{dC_L}{d\alpha}\right)_{\text{in}} - r\Omega\frac{V_\infty + V_{i,z}}{|V_0|}C_D - \frac{(r\Omega)^2}{|V_0|}\frac{dC_D}{d\alpha}\right]$$

Similar expressions for $\frac{dF_z}{du_z}, \frac{dF_y}{du_t}$, etc.

Define $\gamma = dC_L/d\alpha$, $\gamma_e = \sqrt{(dC_L/d\alpha)^2_{\text{in}} + (dC_L/d\alpha)^2_{\text{out}}}$, $\gamma_d = (dC_L/d\alpha)_{\text{in}}$.



Rotationally-sampled isotropic turbulence, axial and tangential components

$$s = \sqrt{(V_\infty \tau)^2 + r_1^2 + r_2^2 - 2r_1 r_2 \cos\Omega\tau}$$

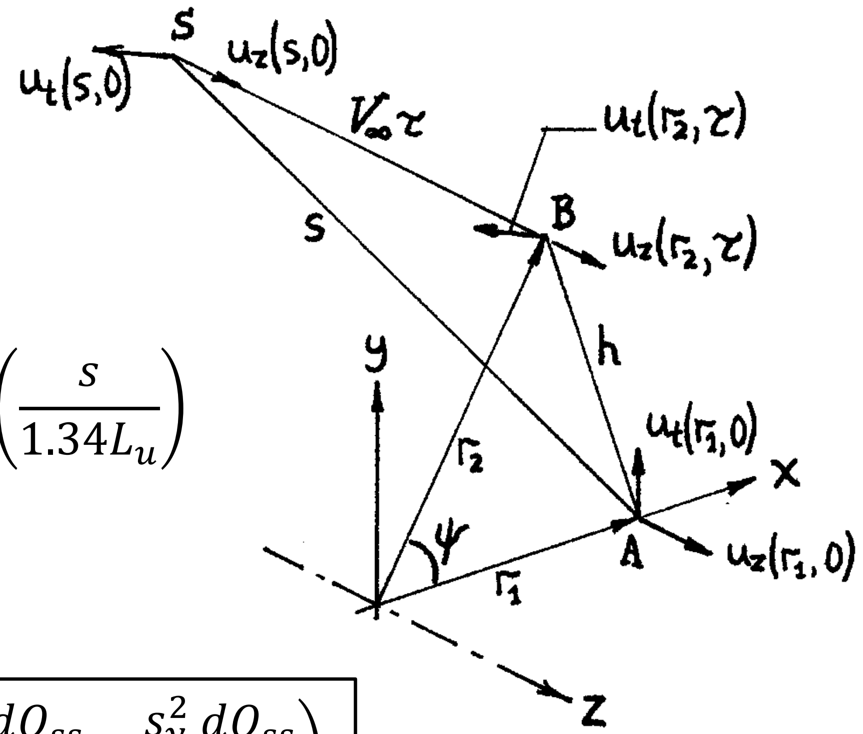
$$Q_{ss} = \frac{2\sigma_u^2}{\Gamma(1/3)} \left(\frac{s}{2.68L_u}\right)^{1/3} K_{1/3}\left(\frac{s}{1.34L_u}\right)$$

$$\frac{dQ_{ss}}{ds} = -\frac{2\sigma_u^2}{\Gamma(1/3)} \left(\frac{1}{1.34L_u}\right)\left(\frac{s}{2.68L_u}\right)^{1/3} K_{-2/3}\left(\frac{s}{1.34L_u}\right)$$

$$Q_{zz} = Q_{ss} + \frac{s}{2}\frac{dQ_{ss}}{ds} - \frac{s_z^2}{2s}\frac{dQ_{ss}}{ds}$$

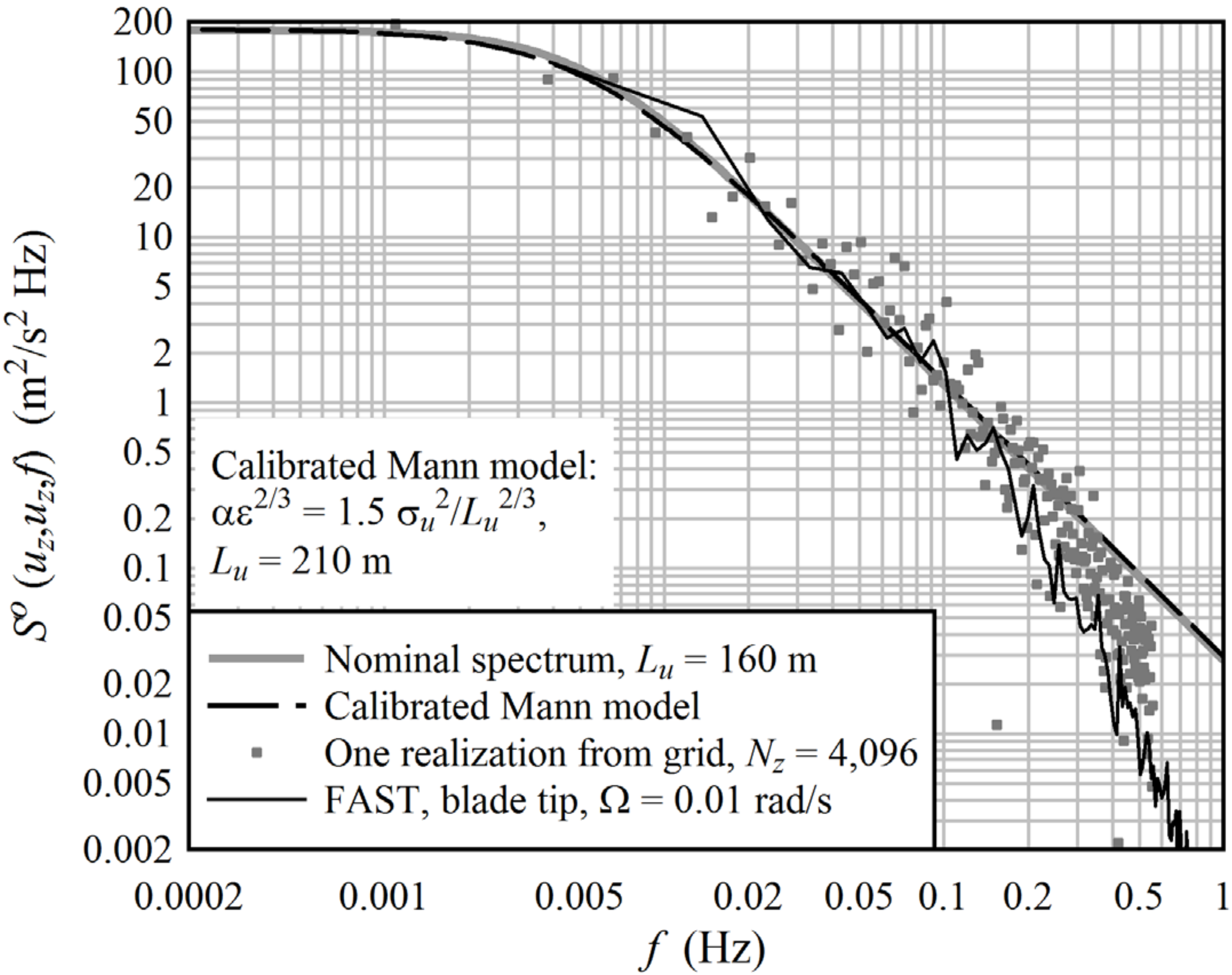
$$\boxed{Q_{tt} = (\sin\Omega\tau)\frac{s_x s_y}{2s}\frac{dQ_{ss}}{ds} + (\cos\Omega\tau)\left(Q_{ss} + \frac{s}{2}\frac{dQ_{ss}}{ds} - \frac{s_y^2}{2s}\frac{dQ_{ss}}{ds}\right)}$$

$$Q_{zt}, Q_{tz} \approx 0$$



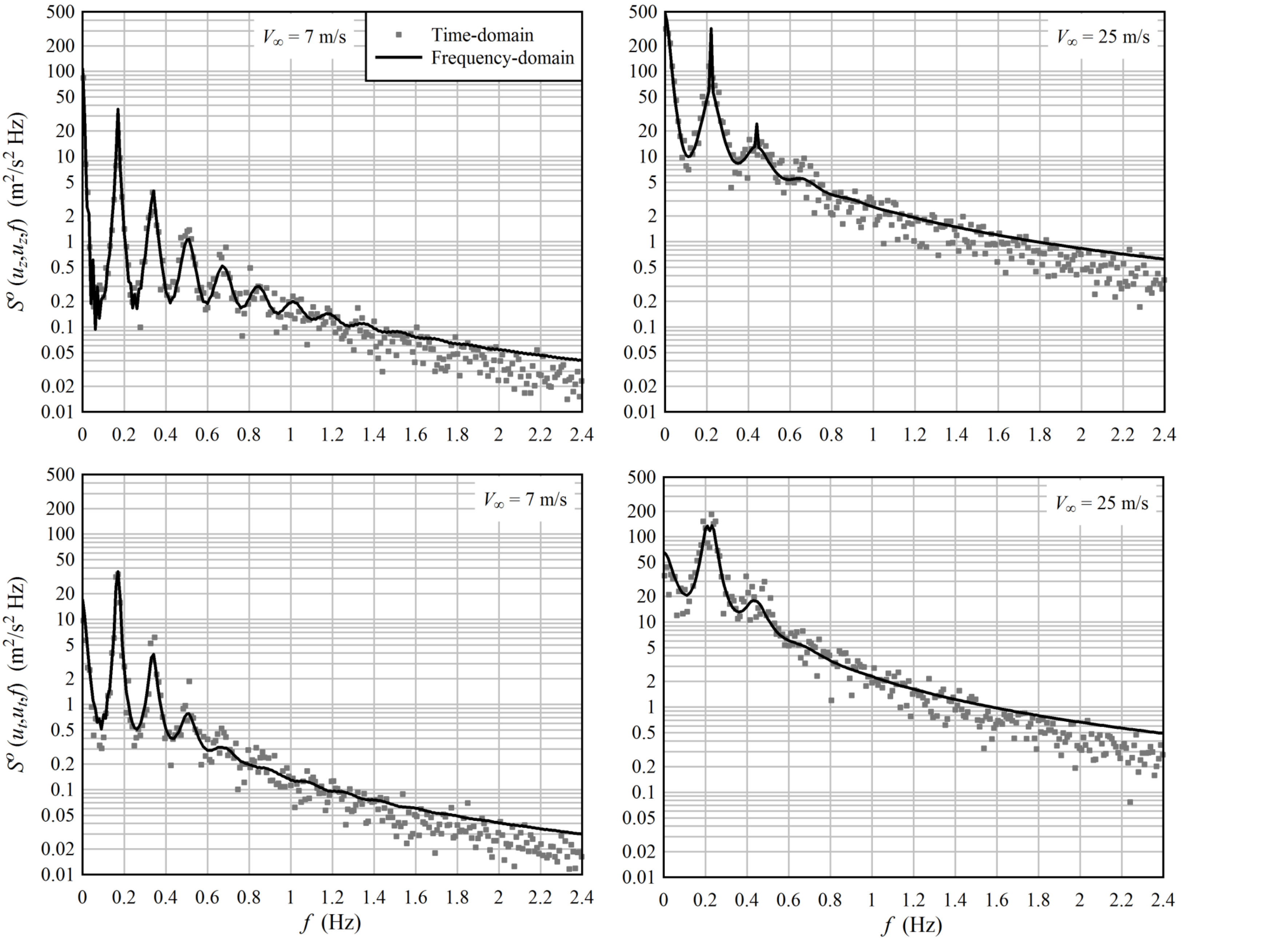
$$Q_{ij} \equiv E\left[u_i(r_1)\, u_j(r_1 + s)\right] = E\left[u_i(r_1,t)\, u_j(r_2, t+\tau)\right]$$

Non-rotational (single-point) turbulence spectrum



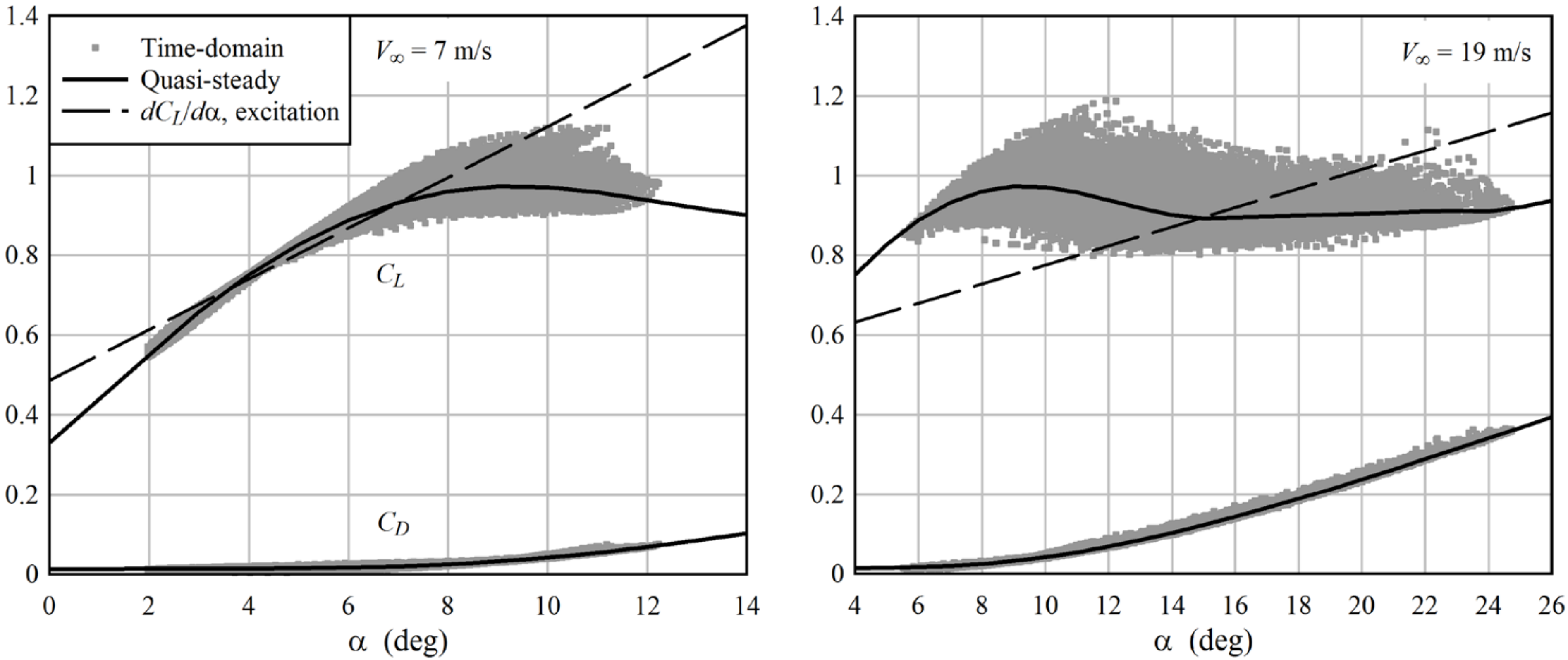
Mann model: Mann, J; "Wind field simulation"; Probabilistic Engineering Mechanics 13 (1998)

Rotationally-sampled turbulence spectrum near blade tip

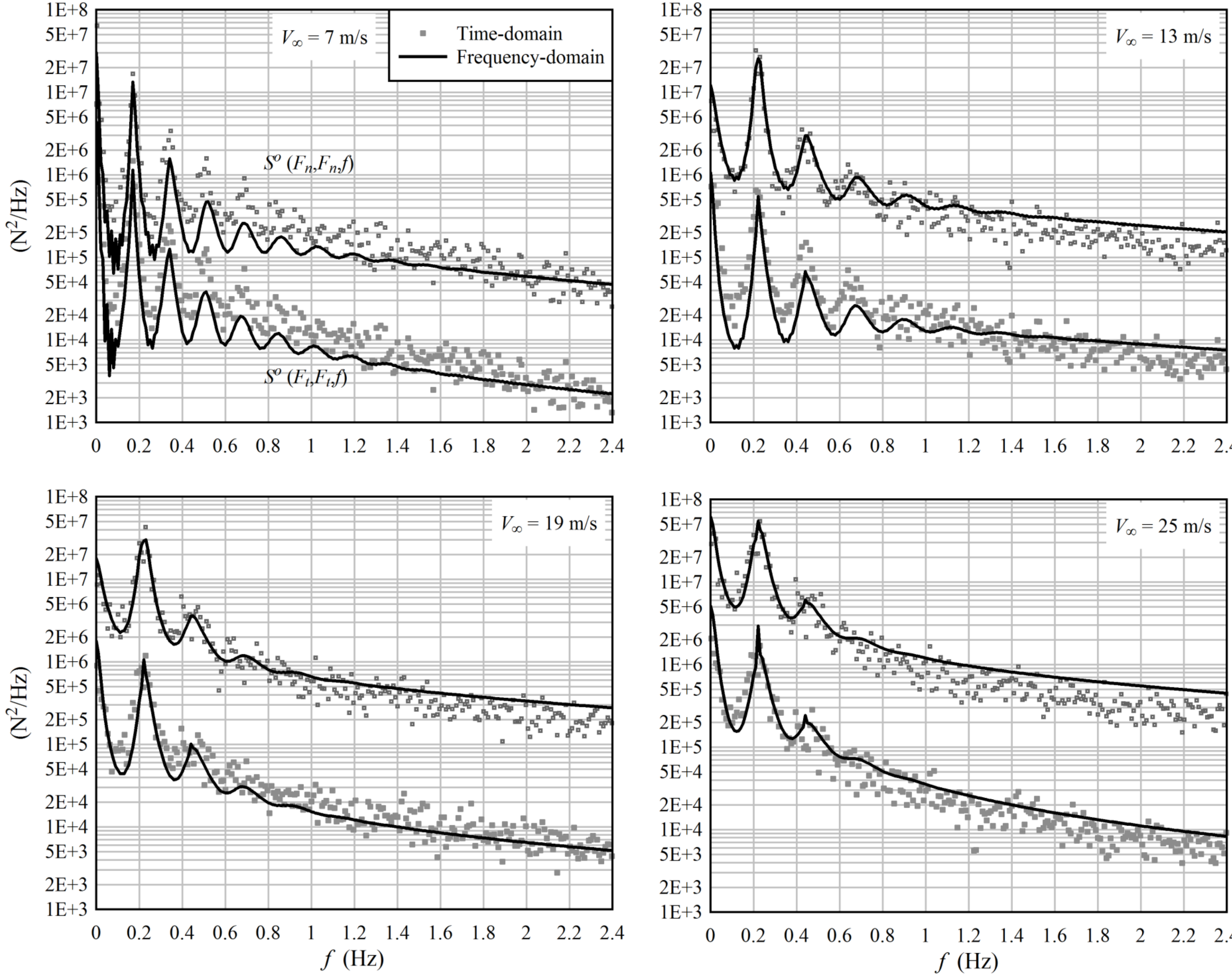


State of a representative outboard airfoil section

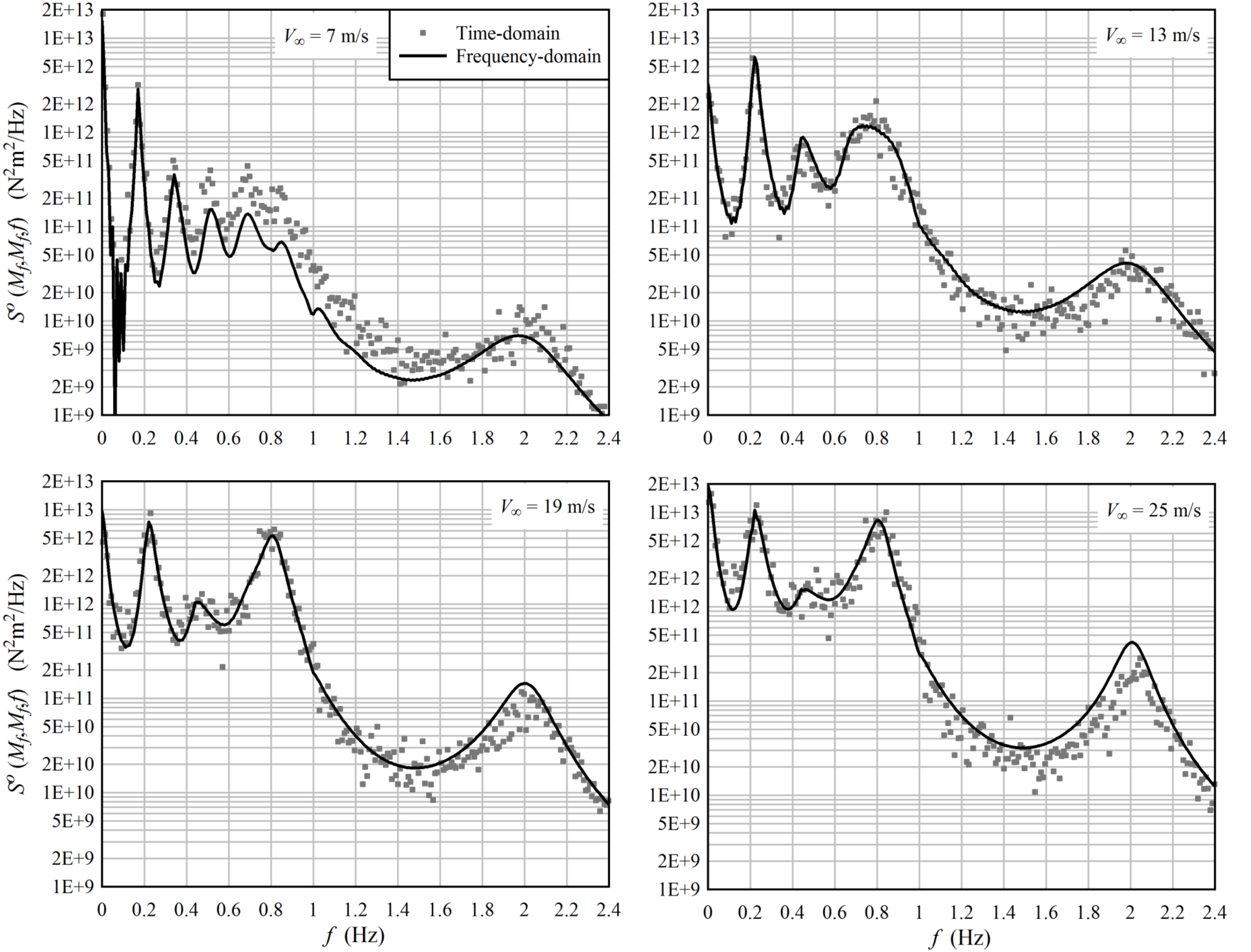
Time-domain: FAST/AeroDyn with Leishman-Beddoes dynamic stall



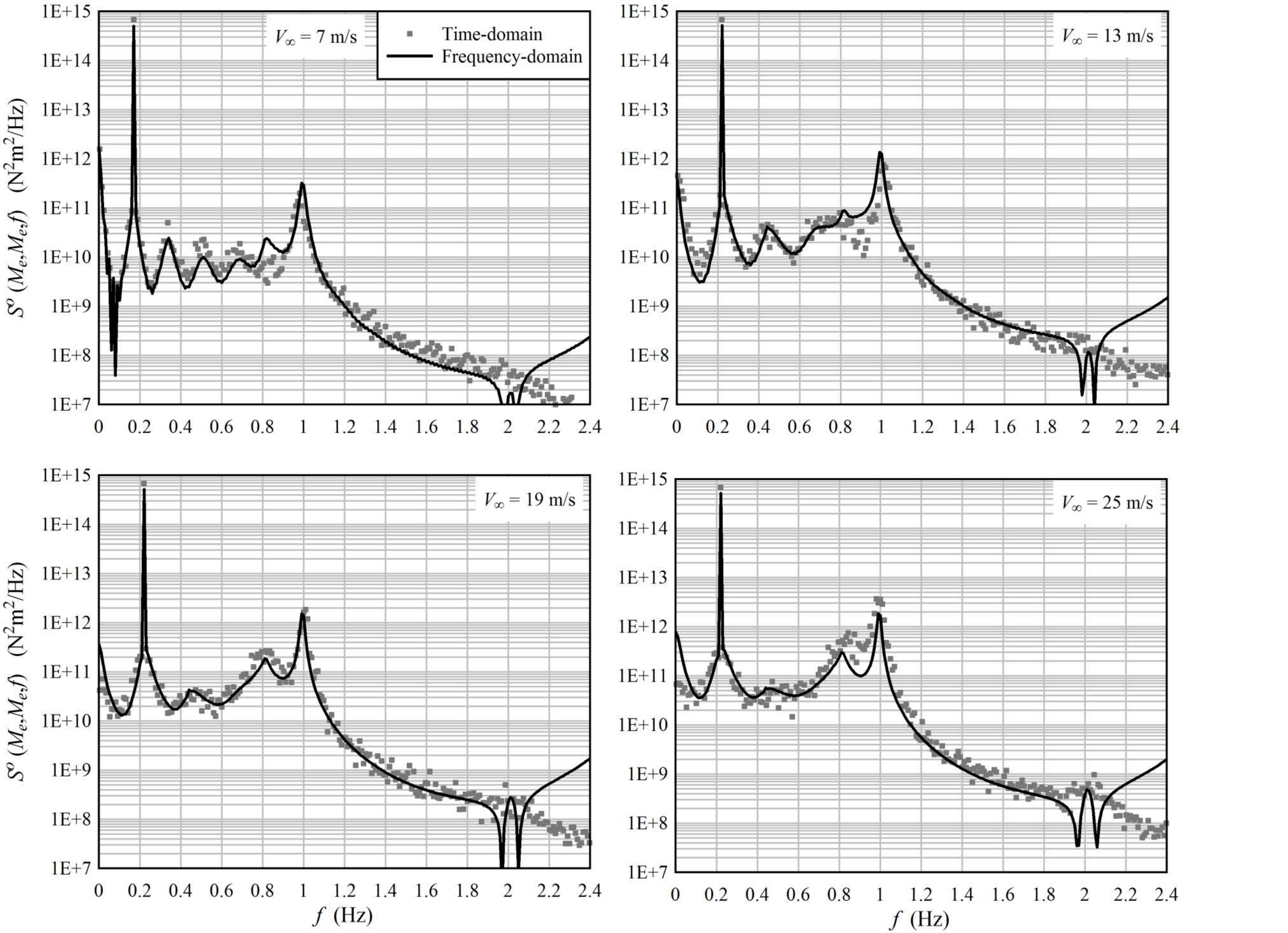
Normal and in-plane aerodynamic loads for an outboard element



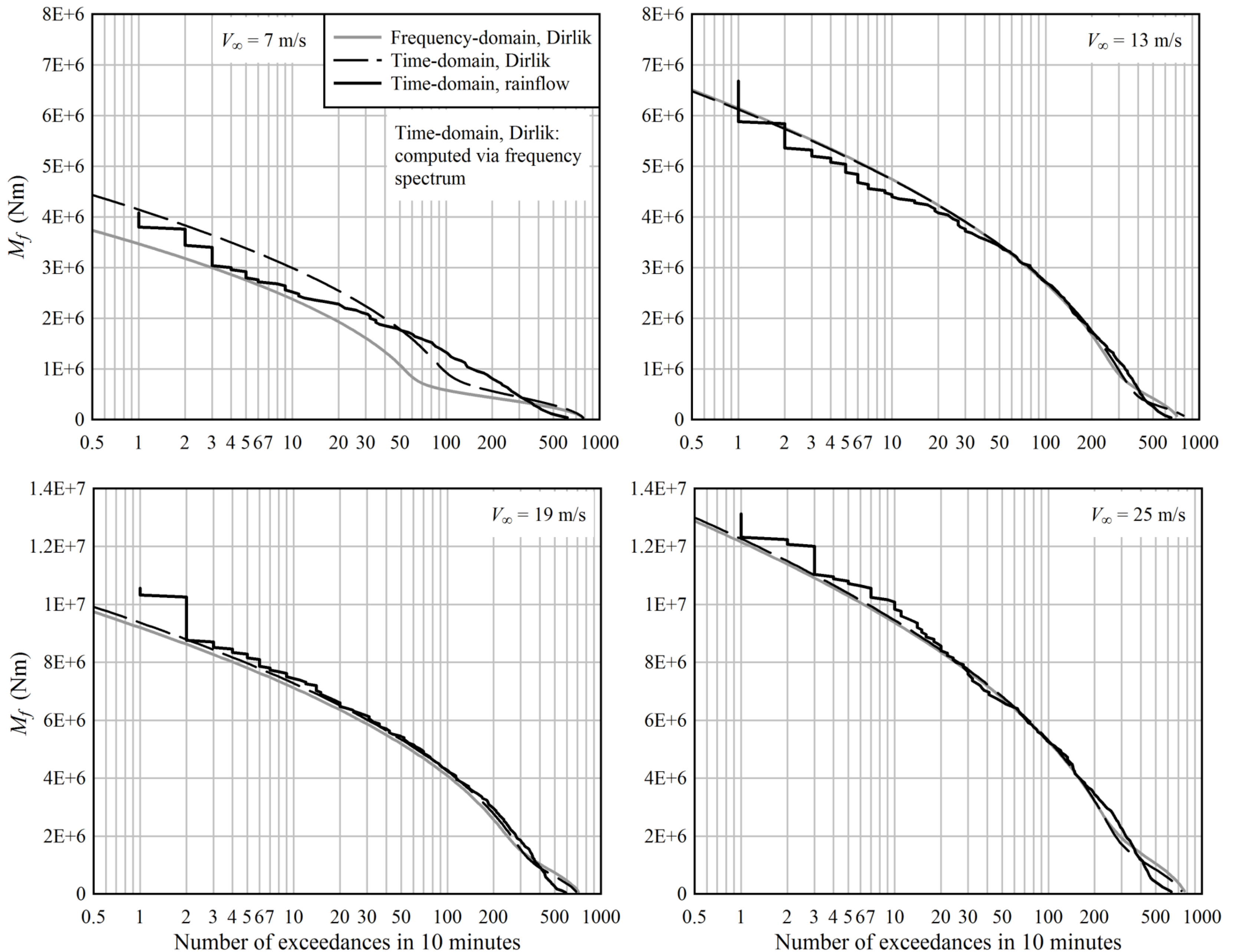
Flapwise blade root bending moments



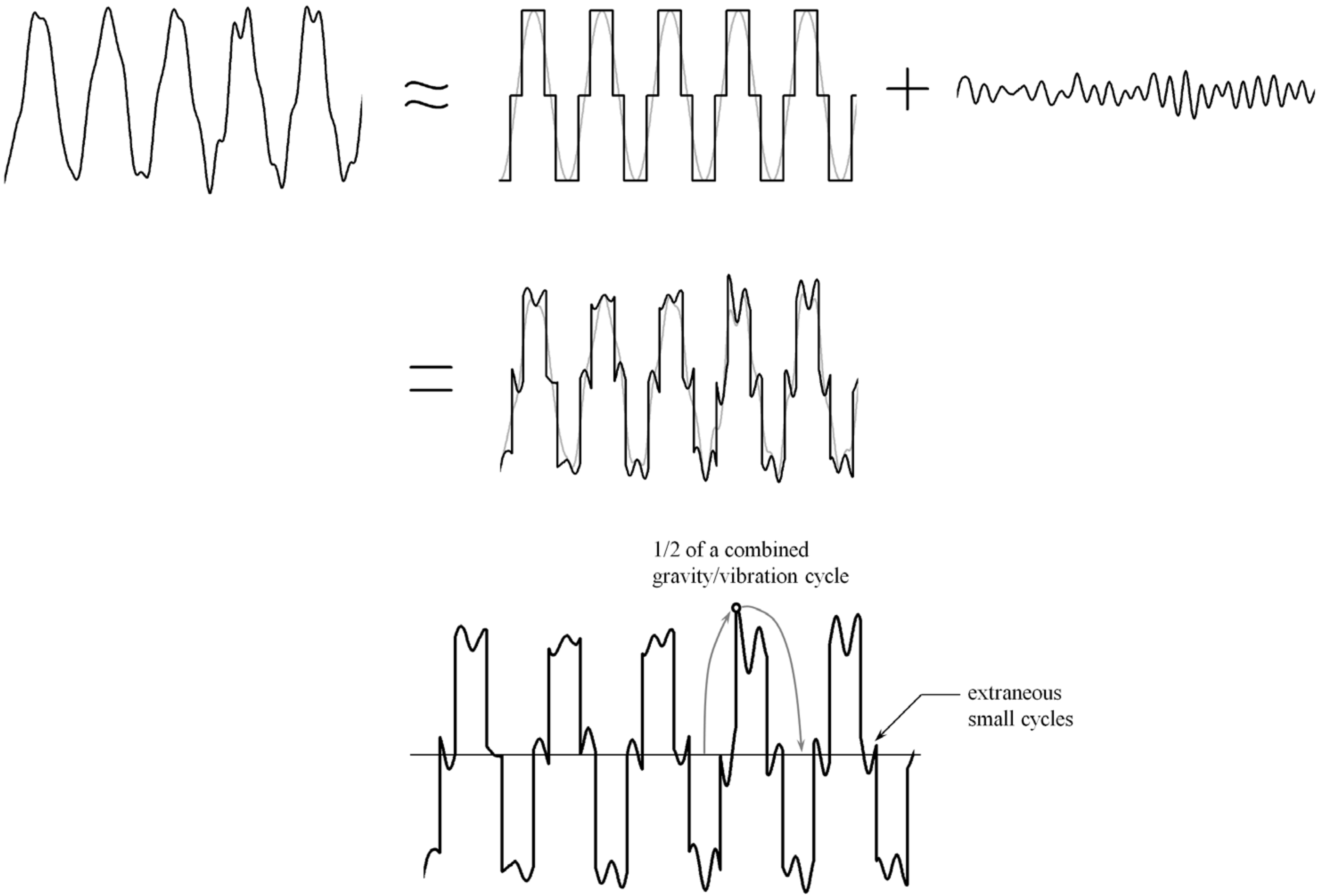
Edgewise blade root bending moments



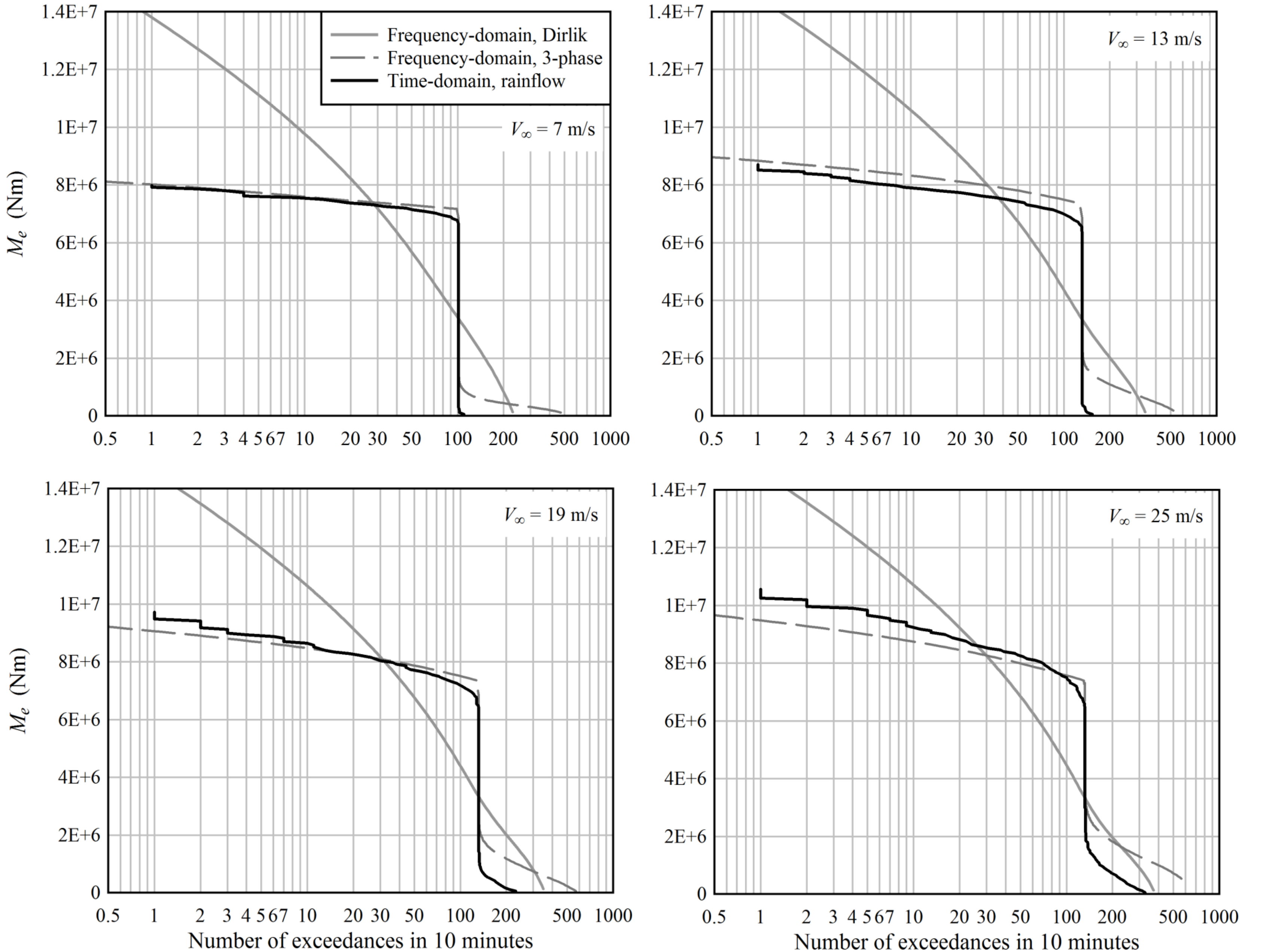
Flapwise blade root bending moment fatigue cycles



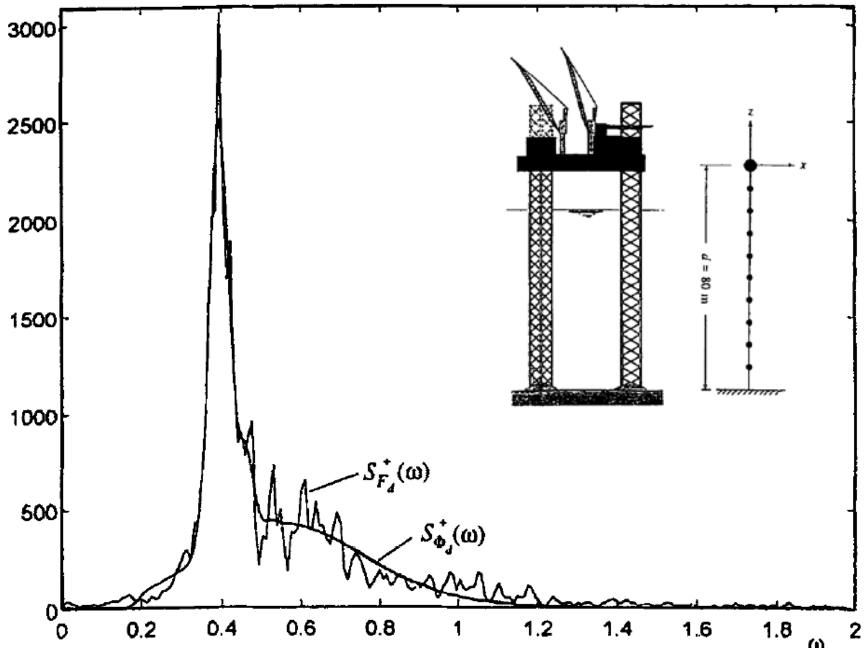
Edgewise blade root bending moment fatigue cycle counting in the frequency domain

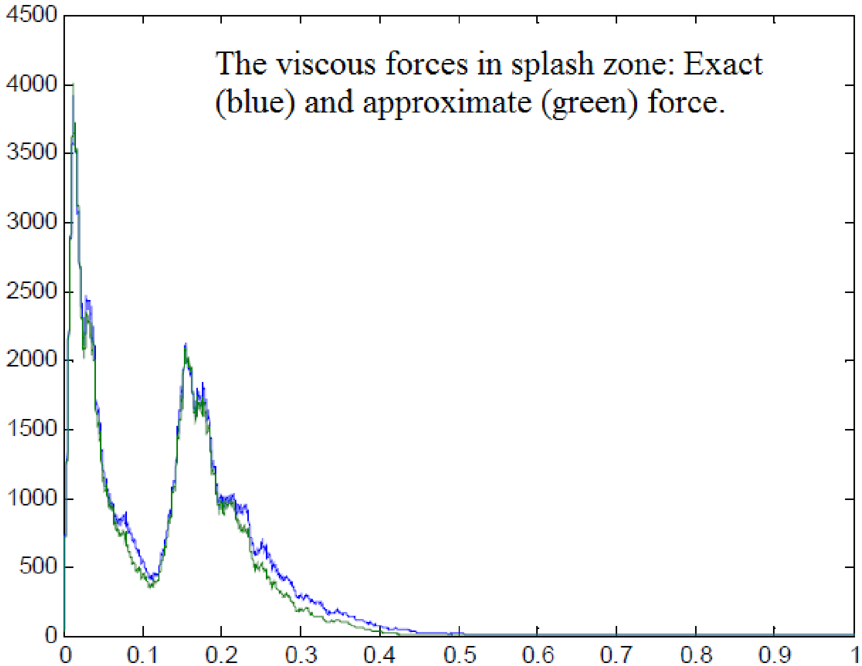


Edgewise blade root bending moment fatigue cycles, including gravity



Wave forces can be linearized stochastically





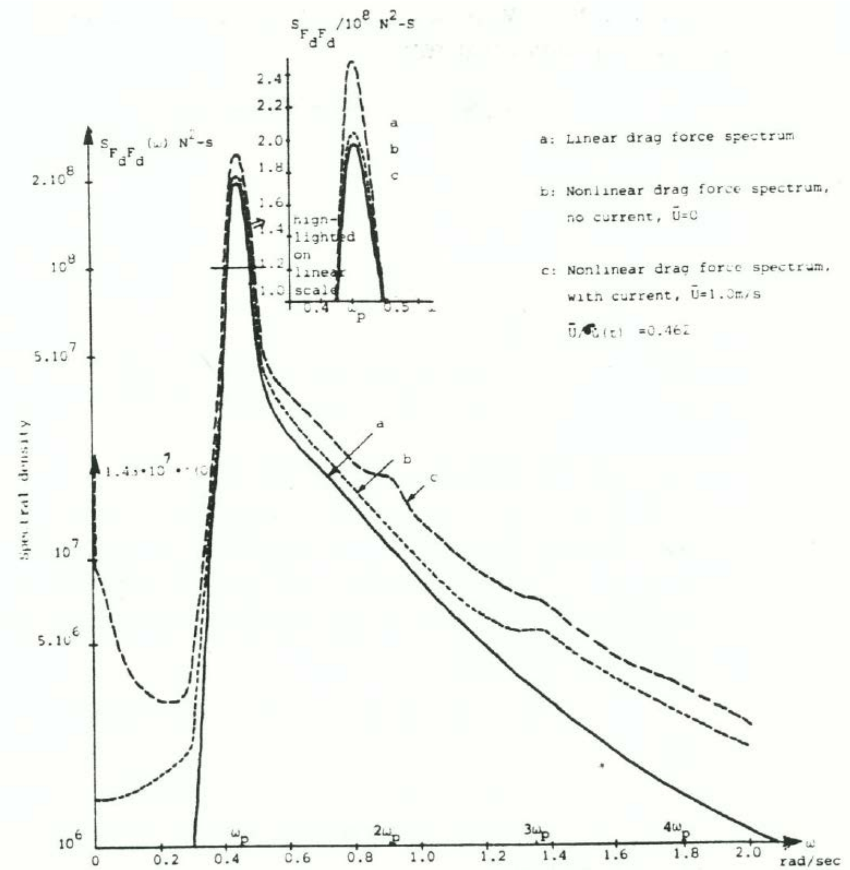
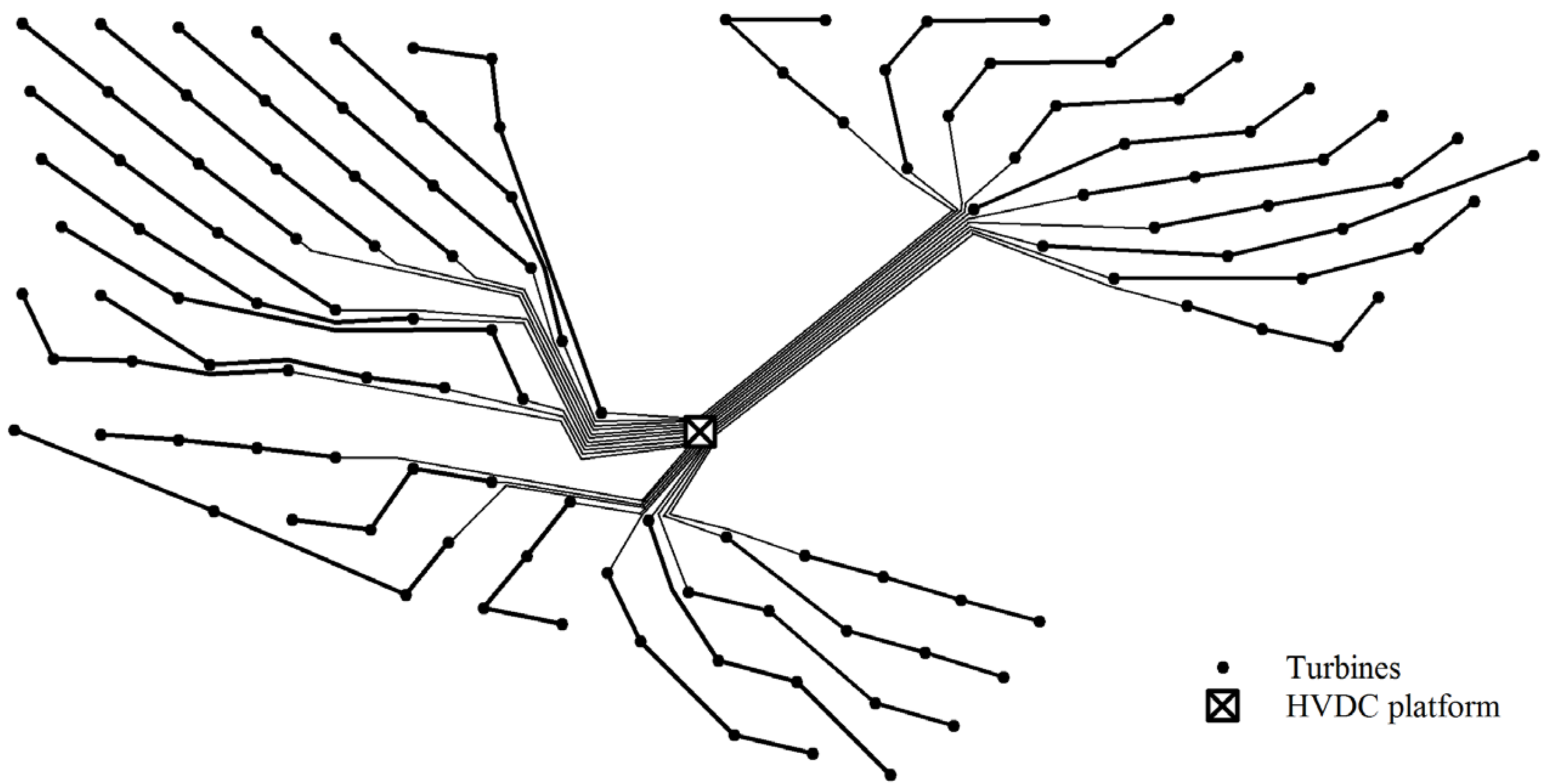


Figure 8. Spectral density function for drag force in the case of a Jonswap state state. $\omega_p = 0.45\,rad/s$, $\gamma = 7$, $\alpha = 0.015$

Upper left: Naess A; Pisano AA; Frequency domain analysis of dynamic response of drag dominated offshore structures; Applied Ocean Research 19 (1997) 251-262. Lower left: Lie H; Kaasen KE; Viscous drift forces on semis in irregular seas – a frequency domain approach; Proceedings OMAE 2008. Above: Gudmestad OT; Connor JJ; Linearization methods and the influence of current on the nonlinear hydrodynamic drag force; Applied Ocean Research 5 (1983) 184-194.

A GW-scale wind power plant

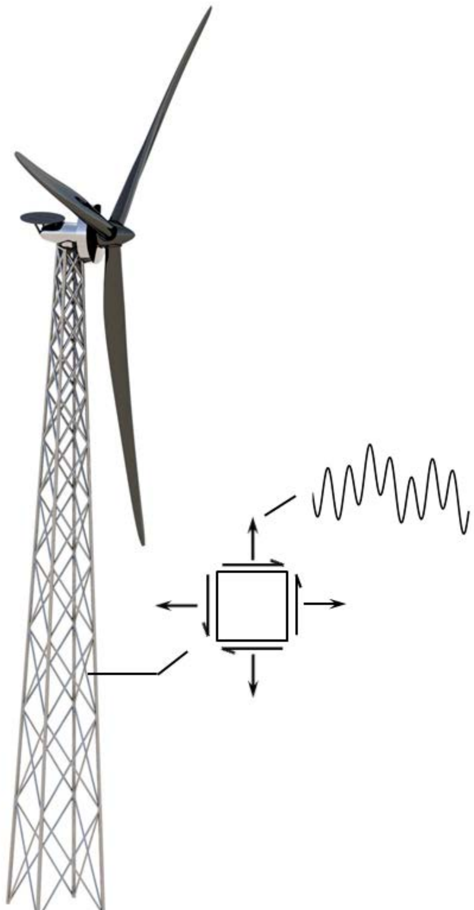


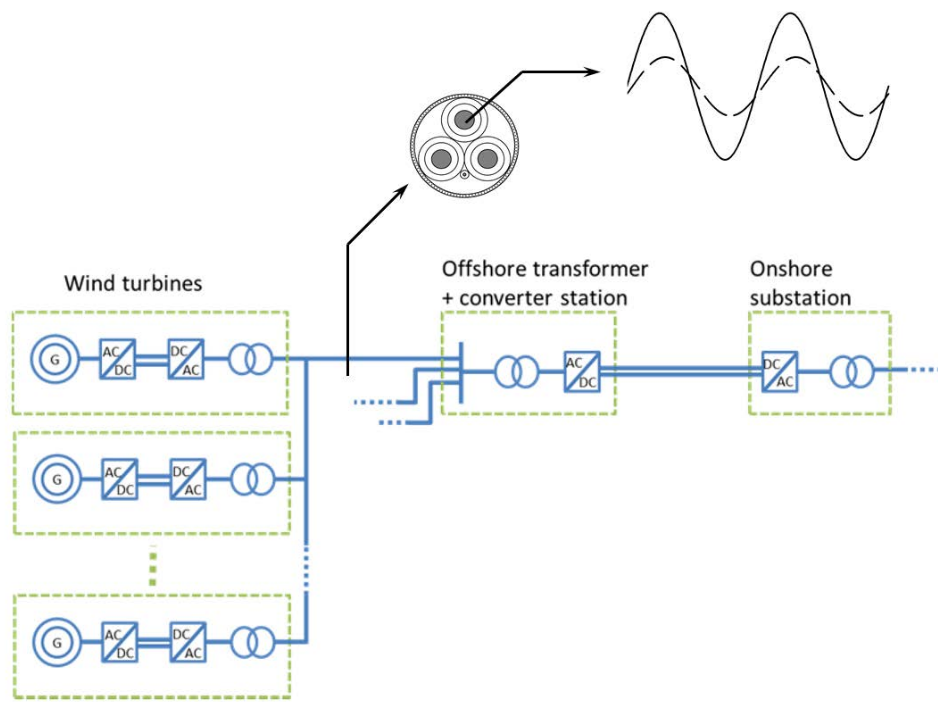
A wind farm as a system

What information do we need from the model?

Probe the structure at any point in space and time: load, deflection; stress, strain
Probe any electrical connection at any time: voltage and current waveforms

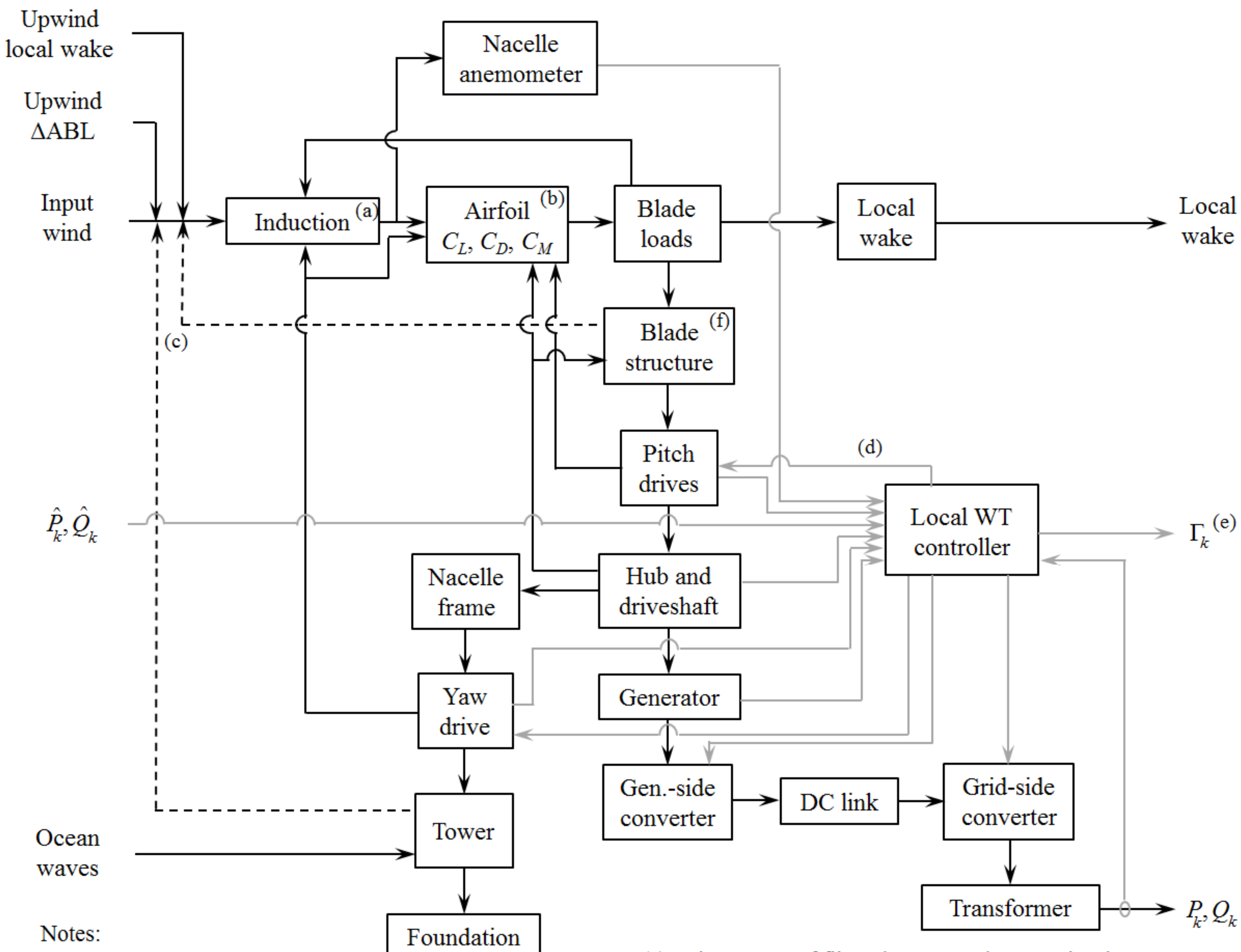
Reduce the amount of information by a stochastic description
(frequency spectra, probability distributions, exceedance curves)





Electric system sketch from: Tande JO; Grid connection of offshore wind farms; Memo AN 12.12.55, Sintef Energy Research, 2012.

A state-space model of a wind farm



Notes:

(a) BEM with dynamic inflow

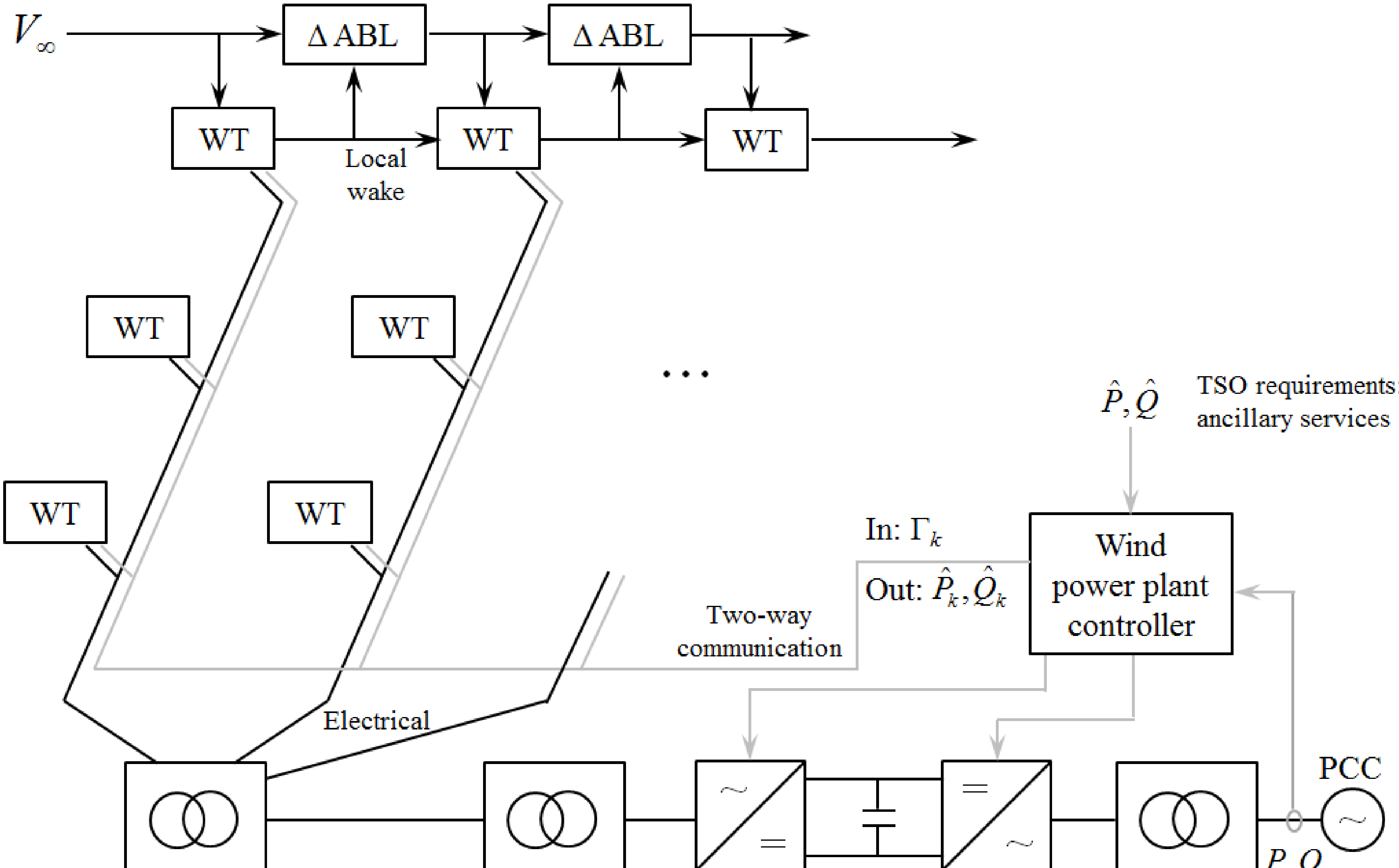
(b) Dynamic stall, Øye type model

(c) Aerodynamic damping, may be linearized and computed independently

(d) Black lines: physical connection. Grey: communication.

(e) Γ is a vector of filtered, processed output signals.

(f) Additional load, strain, or acceleration sensors could be installed in any of the mechanical components and fed to the local controller for active damping or inclusion in the output signal vector Γ.


V_∞
Δ ABL
Δ ABL
WT
WT
WT
Local wake
WT
WT
WT
WT
. . .
$\hat{P}, \hat{Q}$
TSO requirements: ancillary services
In: Γ_k
Out: $\hat{P}_k, \hat{Q}_k$
Wind power plant controller
Two-way communication
Electrical
PCC
P, Q

Number of states per turbine:

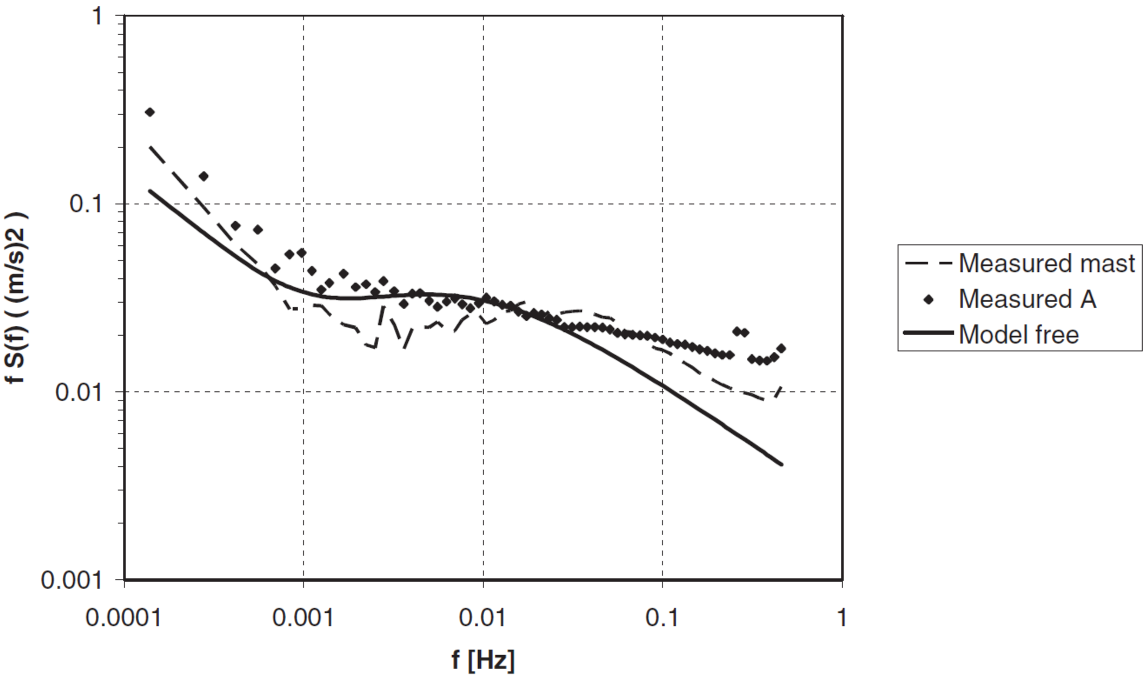
Blade aero:	180	4 dyn inflow, 1 dyn stall per element
Blade structural:	60	10 modes, q, dq/dt, 3 blades
Drivetrain:	20	10 modes
Tower:	20	10 modes

Electrical and control states? Conservatively say it's about the same.

Order of 500 states/turbine, 50000 states for a wind park.

Requires a stochastic wind model based on correlation functions.

Spectral wind field models at the plant scale



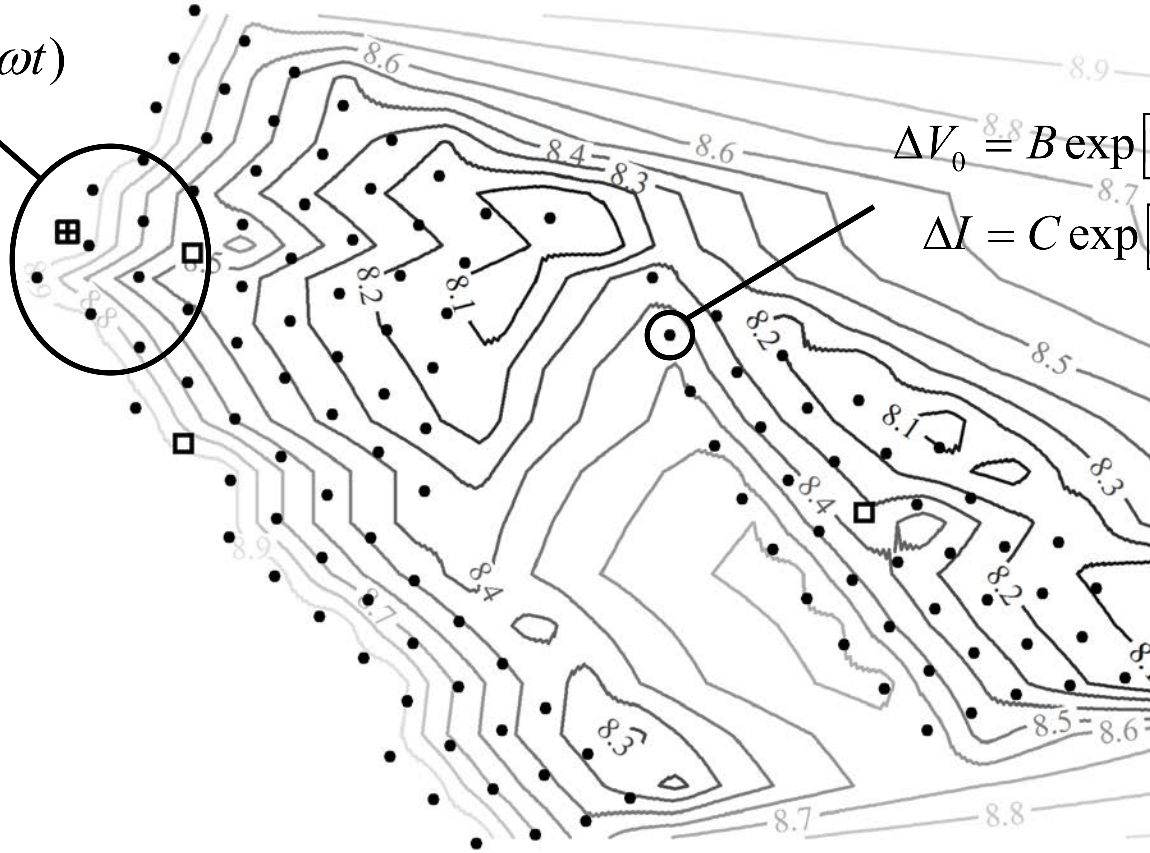
Sørensen P, *et al*. Modelling of power fluctuations from large offshore wind farms. Wind Energy 11 (2008) 29-43.

$$S_{LF}(f) = (\alpha V_\infty)^2 \frac{H/V_\infty}{\left(\frac{H}{V_\infty} f\right)^{5/3} \left(1 + 100\frac{H}{V_\infty} f\right)} \quad \Rightarrow \quad \int_0^{f_1} S_{LF}\, df = \infty ?$$

$\Delta\beta = A\exp(i\omega t)$

$\Delta V_0 = B\exp\left[i\omega\left(t-\alpha\right)\right]$?

$\Delta I = C\exp\left[i\omega\left(t-\beta\right)\right]$?



Within the normal operating range of a wind turbine, can the perturbations be represented by a diffusion function, ideally a linear differential equation?

A relevant experiment: